2019年度 第二回 数理科学 談話会のお知らせ

以下の予定で「2019 年度 第二回 数理科学 談話会」を開催致します。

日　時：2019 年 6 月 20 日（木）16:15–17:15
場　所：弘前大学理工学部 1 号館 5 階 10 番講義室
※講演は英語で行われます (いつもと場所が違いますのでご注意ください)

講演者：Carsten Elsner 氏 (FHDW–University of Applied Sciences, ドイツ)
題目：Diophantine approximation of real and complex numbers restricted by arithmetical conditions

概要: In this talk we consider diophantine approximations for ξ of the type

$$\left| \xi - \frac{p}{q} \right| \leq \frac{C}{|q|^2}\,, \tag{1}$$

where p and q additionally satisfy the arithmetical conditions

$$p \equiv a \pmod{s} \qquad \text{and} \qquad q \equiv b \pmod{s}$$

for fixed numbers a, b, and s. In the case when ξ is a real irrational number and a, b, s are rational integers such that a and b are not both divisible by s, S.Uchiyama in 1980 proved that there are infinitely many pairs of integers p, q satisfying the above inequality (1) with $C = s^2/4$. The author has shown that this constant $C = s^2/4$ is best possible in a general sense.

The main subject of the talk deals with the case when ξ, s, a and b are complex numbers, where s, a, b, p and q belong to the ring of integers K_D of one of the five quadratic imaginary Euclidean fields $\mathbb{Q}(\sqrt{D})$ with $D \in \{-1, -2, -3, -7, -11\}$. Moreover, $\xi \in \mathbb{C} \setminus \mathbb{Q}(\sqrt{D})$ is assumed to be a fixed complex number. We state results of the type (1), namely, for any $D \in \{-1, -2, -3, -7, -11\}$ there exist infinitely many pairs of integers $p, q \in K_D$ satisfying (1) with a suitable positive constant C depending only on D. It turns out that the smallest constant C can be obtained for $D = -3$ using a modified form of a lemma of O.Perron. An outline of the proof for $D = -1$ is sketched, where (1) is expected to hold for infinitely many Gaussian integers p and q.