Diophantine Analysis and Related Fields 2023

Room 11, 2nd Floor, Faculty of Science and Technology Bldg. No. 2,
Bunkyo-cho Campus, Hirosaki University,
Hirosaki, Japan

Abstracts of the Talks

Friday 3rd March

14:30–15:20 **István Pink** (Univ. Debrecen)

Title: **Number of solutions to a special type of unit equations in two unknowns II**

Abstract: This talk is a continuation of the one given by myself on April 17, 2021 at the online Darf seminar. The topic is the best possible general estimate on the number of solutions to a special type of unit equations in two unknowns over the rationals. R. Scott and R. Styer conjectured in 2016 that for any fixed relatively prime positive integers a, b and c greater than 1 the equation $a^x + b^y = c^z$ has at most one solution in positive integers x, y and z, except for specific cases. In this talk we give a brief introduction to the conjecture, and present our results with their proofs, which in particular provides an analytic proof of the celebrated theorem of Scott (1993) solving the conjecture for $c = 2$ in a purely algebraic manner. This is a joint work with Takafumi Miyazaki (Gunma University).

15:40-16:30 **Koichi Kawada** (Iwate Univ.)

Title: **An overview of the Waring-Goldbach problem**

Abstract: The talk begins with an overview of the current state of research on representations of numbers by sums of powers of primes, for general larger exponents. Then I turn to the cases of smaller exponents, especially sums of cubes of primes, and talk on the result obtained jointly with Lilu Zhao, that every sufficiently large even number can be expressed as $x^3 + p_1^3 + p_2^3 + \cdots + p_7^3$ with primes p_i and a P_2-number x.

16:50-17:30 **Yusuke Washio** (Nihon Univ.)

Title: **A linear independence measure of polylogarithms relying on Padé approximation**

Abstract: We obtain a linear independence measure related to polylogarithms and values of Lerch functions at algebraic points, by using Padé approximation. We also talk about new examples of independence measure concerning with the criterion obtained by S. David (Sorbonne Univ.), N. Hirata-Kohno (CST, Nihon Univ.) and M. Kawashima (CIT, Nihon Univ.). This is a joint work with Ryuji Muroi (CST, Nihon Univ.).

Saturday 4th March

9:30-10:20 **Daniel Duverney** (Baggio Engineering School)

Title: **Sylvester sequence, irrationality and transcendence**

Abstract: Let $(S_n)_{n\geq 0}$ be the Sylvester sequence, defined by

$$S_0 = 2, \quad S_{n+1} = S_n^2 - S_n + 1 \quad (n \geq 0)\,.$$

The purpose of this talk is to present some results on the irrationality and transcendence of the numbers

$$F=\sum_{n=0}^{\infty}\frac{a_n}{S_n+b_n}$$
$$G=\prod_{n=0}^{\infty}\left(1+\frac{a_n}{S_n+b_n}\right),\quad S_n+a_n+b_n\neq 0\ \ (n\geq 0)$$
$$H=\frac{a_0}{S_0+b_0+}\,\frac{a_1}{S_1+b_1+}\cdots+\frac{a_n}{S_n+b_n+}\cdots,$$

where a_n and b_n are integers satisfying

$$S_n+b_n\neq 0\ \ (n\geq 0)\qquad \text{and}\qquad \log\left(\max\{1,|a_n|,|b_n|\}\right)=o\left(2^n\right).$$

We can solve completely the problem of the irrationality of F, G and H. We can also prove that the continued fraction H is transcendental by using Roth theorem. For the transcendence of F and G (for which Mahler's method seems to be the best tool), we can prove it only in very special cases. This talk is based in part on joint works with Takeshi Kurosawa and Iekata Shiokawa.

10:40-11:30 **Dong Han Kim** (Dongguk Univ.)

Title: **The Markoff spectrum on the Hecke group**

Abstract: We consider the Markoff spectrum on the Hecke group. The Markoff spectrum on H4 is known as the Markoff spectrum of index 2 sublattices by Vulakh and the Markoff spectrum of 2-minimal forms or C-minimal forms by Schmidt, who characterized the spectrum up to the first accumulation point. The Markoff spectrum on H6 is also known as the Markoff spectrum of 3-minimal forms by Schmidt. After the first accumulation point, we show that both spectra have positive Hausdorff dimension and find gaps. This talk is based on joint work with Byungchul Cha and Deokwon Sim.

11:50-12:30 **Haruki Ide** and **Taka-aki Tanaka** (Keio Univ.)

Title: **Mahler's method for algebraic independence of partial derivatives of certain series in several variables**

Abstract: Some entire functions are known to have the properties that all the values of successive derivatives of each of those functions at any nonzero algebraic numbers are algebraically independent. Examples of such entire functions were shown independently by Nishioka and the second speaker in the case of one variable and in the case of two variables by the first speaker. In this talk we construct an entire function of arbitrary number of variables having the following property: The infinite set consisting of all the values of all its partial derivatives of any orders at all algebraic points, including zero components, is algebraically independent. One of the key points in the proof of this result is to find a biregular transformation between the partial derivatives of the entire function in question and those of a certain Lambert type series in several variables. Using this biregular transformation, we reduce the problem to the algebraic independency of the values of the partial derivatives of the Lambert type series, which can be represented as special values of certain Mahler functions. Another key point in the proof is to prove the linear independency of the Mahler functions themselves, for which we introduce a new method.

14:30-15:20 **Damien Jamet** (Univ. Lorraine)

Title: **On a probabilistic extension of the Oldenburger-Kolakoski sequence**

Abstract: The Oldenburger-Kolakoski sequence is the only infinite sequence over the alphabet $\{1, 2\}$ that starts with 1 and is its own run-length encoding. In the present work, we take a step back from this largely known and studied sequence by introducing some randomness in the choice of the letters written. This enables us to provide some results on the convergence of the density of 1's in the resulting sequence. When the choice of the letters is given by an infinite sequence of i.i.d. random variables or by a Markov chain, the average densities of letters converge. Moreover, in the case of i.i.d. random variables, we are able to prove that the densities even almost surely converge.

15:40-16:20 **Hiroaki Ito** (Univ. Tsukuba)

Title: **Symmetry of continued fraction algorithms**

Abstract: F. Schweiger introduced a fibred system of multidimensional continued fraction algorithms. An advantage of a fibred system is that it often provides a systematic construction of absolutely continuous invariant measure by using a dual system. We can find a symmetry in measure by investigating the dual algorithms, I will talk about the symmetry of several algorithms. Also, I present examples with partial self-dual and examples that are ergodic but not self-dual.

16:40-17:20 **Kota Saito** (Univ. Tsukuba)

Title: **Finiteness of solutions of Diophantine equations on Piatetski-Shapiro sequences**

Abstract: A sequence of positive integers of the form $\lfloor n^\alpha \rfloor$ for some non-integral $\alpha > 1$ is called a Piatetski-Shapiro sequence, for short PS-sequence. Let $\mathrm{PS}(\alpha) = \{\lfloor n^\alpha \rfloor : n = 1, 2, \ldots\}$. In this talk, we discuss the linear equations (E) $x + y = z$ on $\mathrm{PS}(\alpha)$. As a main result, we show that for almost all $\alpha > 3$, (E) has at most finitely many solutions $(x, y, z) \in \mathrm{PS}(\alpha)$. If time permitted, we give a result for general linear equations $y = a_1x_1 + \cdots + a_nx_n$ and the Hausdorff dimension.

Sunday 5th March

9:30-10:20 **Pierre Popoli** (Univ. Lorraine)

Title: **On the binary digits of n and n^2**

Abstract: Let $s(n)$ denote the sum of digits in the binary expansion of the integer n. Hare, Laishram and Stoll (2011) studied the number of odd integers such that $s(n) = s(n^2) = k$, for a given positive integer k. The remaining cases that could not be treated by theses authors were $k = 9, 10, 11, 14$ or 15. In this talk, I will present the results of our article on the cases $k = 9, 10$ and 11 and the difficulties to settle for the two remaining cases $k = 14$ and 15. A related problem is to study perfect squares of odd integers with four binary digits. Bennett, Bugeaud and Mignotte (2012) proved that there are only finitely many solutions and conjectured that the set of solutions is composed of $13, 15, 47$ and 111. In the same paper, we give an algorithm to find all solutions with fixed sum of digits value, supporting this conjecture, as well as show related results for perfect squares of odd integers with five binary digits. This is joint work with Aloui, Jamet, Kaneko, Kopecki and Stoll.

10:40-11:30 **Makoto Kawashima** (Nihon Univ.)

Title: **On arithmetic properties of hypergeometric functions**

Abstract: The classical hypergeometric functions, for instance Gauss hypergeometric function, are familiar in number theory and many other areas of applications. In this talk, we shall discuss the arithmetic properties of these functions and their values. In particular, we will report our recent results on the linear independence of values of contiguous hypergeometric functions at distinct points. This is a joint work with S. David and N. Hirata-Kohno.

11:50-12:30 **Fan Wen** (Univ. Tsukuba)

Title: **Hausdorff dimension on Markoff-Lagrange problem**

Abstract: We focus on a set $\mathcal{L}(\alpha)$ called Lagrange spectrum of geometric progressions, the definition of $\mathcal{L}(\alpha)$ is as follow:

$$\mathcal{L}(\alpha) = \left\{ \limsup_{n\to\infty} ||\xi\alpha^n|| \;\middle|\; \xi \in \mathbb{R} \right\},$$

where $||\xi\alpha^n||$ is the distance from $\xi\alpha^n$ to the nearest integer.

Some properties of $\mathcal{L}(\alpha)$ have been discussed in the history. And what we want to do is estimating the Hausdorff dimension of set $\mathcal{L}(\alpha) \cap [0,t]$ where $t \in [0, \frac{1}{2})$. In the case that $\alpha \geq 2$ is an integer, we give a set $F(t)$ which contains $\mathcal{L}(\alpha) \cap [0,t]$. And we obtain some properties of $F(t)$ and give the upper bounds of $\dim_H(F(t))$.