2017 年度 第三回 数理科学 談話会のお知らせ

以下の予定で「2017 年度 第三回 数理科学 談話会」を開催致します。

日時：2017 年 6 月 1 日（木）16:10–17:10
場所：弘前大学理工学部 2 号館 2 階 11 番講義室
(いつもと場所が違いますのでご注意ください)

講演者：Carsten Elsner 氏 (FHDW–University of Applied Sciences, ドイツ)
題目：On Error Sums
概要: Let p_n/q_n $(n \geq 0)$ denote the n-th convergent of the continued fraction expansion of a real number α. During the last years a lot of studies are concerned with series formed by the error terms $q_n\alpha - p_n$. A surprising result on such error sums is the identity

$$\int_0^1 \sum_{m\geq 0} |q_m\alpha - p_m| \, d\alpha = \frac{3\zeta(2)\log 2}{2\zeta(3)} - \frac{5}{8} = 0.79778798\ldots$$

showing the mean of the integrable error sum function between 0 and 1. Similar explicit expressions are available in general for the integrals

$$I_n := \int_0^1 \sum_{m\geq 0} |q_m\alpha - p_m|^n \, d\alpha \qquad (n \geq 1)\,.$$

It turns out that I_1, I_2, I_3, I_4 are algebraically dependent over $\mathbb{Q}$.
In the first part of the talk we consider particular error sums with denominators q_m restricted to arithmetic progressions. We focus our attention to error sums of the form

$$\sum_{q_m \equiv l \pmod{k}} |q_m\alpha - p_m|\,.$$

These functions are Riemann-integrable from $\alpha = 0$ to $\alpha = 1$ as well. Their values are expressible by so-called multiple sums, from which lower and upper bounds can be obtained. Moreover, under some restrictions on k and l, asymptotic formulas for the integrals are available.
The second part of the talk is devoted to the generating function

$$\mathcal{E}(\alpha, t) := \sum_{n\geq 0} t^n |q_n\alpha - p_n|\,.$$

A variant is given by the error sum function

$$\mathcal{E}_{MC}(\alpha, t) := |\alpha - a_0| + \sum_{\nu=1}^{\infty} t^\nu \sum_{1\leq b\leq a_\nu} \big|(bq_{\nu-1} + q_{\nu-2})\alpha - (bp_{\nu-1} + p_{\nu-2})\big|$$

which additionally takes into account all the minor convergents of α. Both the functions, $\mathcal{E}(\alpha, t)$ and $\mathcal{E}_{MC}(\alpha, t)$, are investigated taking analytic and arithmetic aspects into account. We place the main focus on certain values α given by the exponential function.